Tennessee Math Standards

Introduction

The Process

The Tennessee State Math Standards were reviewed and developed by Tennessee teachers for Tennessee schools. The rigorous process used to arrive at the standards in this document began with a public review of the then-current standards. After receiving 130,000+ reviews and 20,000+ comments, a committee composed of Tennessee educators spanning elementary through higher education reviewed each standard. The committee scrutinized and debated each standard using public feedback and the collective expertise of the group. The committee kept some standards as written, changed or added imbedded examples, clarified the wording of some standards, moved some standards to different grades, and wrote new standards that needed to be included for coherence and rigor. From here the standards went before the appointed Standards Review Committee to make further recommendations before being presented to the Tennessee Board of Education for final adoption.

The result is Tennessee Math Standards for Tennessee Students by Tennesseans.

Mathematically Prepared

Tennessee students have various mathematical needs that their K-12 education should address.

All students should be able to recall and use their math education when the need arises. That is, a student should know certain math facts and concepts such as the multiplication table, how to add, subtract, multiply, and divide basic numbers, how to work with simple fractions and percentages, etc. There is a level of procedural fluency that a student's K-12 math education should provide him or her along with conceptual understanding so that this can be recalled and used throughout his or her life. Students also need to be able to reason mathematically. This includes problem solving skills in work and non-work related settings and the ability to critically evaluate the reasoning of others.

A student's K-12 math education should also prepare him or her to be free to pursue post-secondary education opportunities. Students should be able to pursue whatever career choice, and its post-secondary education requirements, that they desire. To this end, the K-12 math standards lay the foundation that allows any student to continue further in college, technical school, or with any other post-secondary educational needs.

A college and career ready math class is one that addresses all of the needs listed above. The standards' role is to define what our students should know, understand, and be able to do mathematically so as to fulfill these needs. To that end, the standards address conceptual understanding, procedural fluency, and application.

Conceptual Understanding, Procedural Fluency, and Application

In order for our students to be mathematically proficient, the standards focus on a balanced development of conceptual understanding, procedural fluency, and application. Through this balance, students gain understanding and critical thinking skills that are necessary to be truly college and career ready.

Conceptual understanding refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.

Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly. One cannot stop with memorization of facts and procedures alone. It is about recognizing when one strategy or procedure is more appropriate to apply than another. Students need opportunities to justify both informal strategies and commonly used procedures through distributed practice. Procedural fluency includes computational fluency with the four arithmetic operations. In the early grades, students are expected to develop fluency with whole numbers in addition, subtraction, multiplication, and division. Therefore, computational fluency expectations are addressed throughout the standards. Procedural fluency extends students' computational fluency and applies in all strands of mathematics. It builds from initial exploration and discussion of number concepts to using informal strategies and the properties of operations to develop general methods for solving problems (NCTM, 2014).

Application provides a valuable context for learning and the opportunity to practice skills in a relevant and a meaningful way. As early as Kindergarten, students are solving simple "word problems" with meaningful contexts. In fact, it is in solving word problems that students are building a repertoire of procedures for computation. They learn to select an efficient strategy and determine whether the solution(s) makes sense. Problem solving provides an important context in which students learn about numbers and other mathematical topics by reasoning and developing critical thinking skills (Adding It Up, 2001).

Progressions

The standards for each grade are not written to be nor are they to be considered as an island in and of themselves. There is a flow, or progression, from one grade to the next, all the way through to the high school standards. There are four main progressions that are composed of mathematical domains/conceptual categories (see the Structure section below and color chart on the following page).

The progressions are grouped as follows:

Grade	Domain/Conceptual Category
K	Counting and Cardinality
K-5	Number and Operations in Base Ten
3-5	Number and Operations – Fractions
6-7	Ratios and Proportional Relationships
6-8	The Number System
9-12	Number and Quantity
K-5	Operations and Algebraic Thinking
6-8	Expressions and Equations
8	Functions
9-12	Algebra and Functions
K-12	Geometry
K-5	Measurement and Data
6-12	Statistics and Probability

State Standards – Mathematics Learning Progressions

<table>
<tr><th>Kindergarten</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="2">Ratios and Proportional Relationships</td><td></td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations - Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="6">Geometry</td><td colspan="3">Geometry</td><td>Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics and Probability</td></tr>
</table>

Each of the progressions begins in Kindergarten, with a constant movement toward the high school standards as a student advances through the grades. This is very important to guarantee a steady, age appropriate progression which allows the student and teacher alike to see the overall coherence of and connections among the mathematical topics. It also ensures that gaps are not created in the mathematical education of our students.

Structure of the Standards

Most of the structure of the previous state standards has been maintained. This structure is logical and informative as well as easy to follow. An added benefit is that most Tennessee teachers are already familiar with it.

The structure includes:

- **Content Standards -** Statements of what a student should know, understand, and be able to do.

- **Clusters -** Groups of related standards. Cluster headings may be considered as the big idea(s) that the group of standards they represent are addressing. They are therefore useful as a quick summary of the progression of ideas that the standards in a domain are covering and can help teachers to determine the focus of the standards they are teaching.

- **Domains -** A large category of mathematics that the clusters and their respective content standards delineate and address. For example, *Number and Operations – Fractions* is a domain under which there are a number of clusters (the big ideas that will be addressed) along with their respective content standards, which give the specifics of what the student should know, understand, and be able to do when working with fractions.

- **Conceptual Categories –** The content standards, clusters, and domains in the 9^{th}-12^{th} grades are further organized under conceptual categories. These are very broad categories of mathematical thought and lend themselves to the organization of high school course work. For example, Algebra is a conceptual category in the high school standards under which are domains such as Seeing Structure in Expressions, Creating Equations, Arithmetic with Polynomials and Rational Expressions, etc.

Standards and Curriculum

It should be noted that the standards are what students should know, understand, and be able to do; but, they do not dictate how a teacher is to teach them. In other words, the standards do not dictate curriculum. For example, students are to understand and be able to add, subtract, multiply, and divide fractions according to the standards. Although within the standards algorithms are mentioned and examples are given for clarification, how to approach these concepts and the order in which the standards are taught within a grade or course are all decisions determined by the local district, school, and teachers.

Example from the Standards' Document for K – 8

Taken from 3rd Grade Standards:

Measurement and Data (MD)	
Cluster Headings	**Content Standards**
A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve contextual problems involving addition and subtraction of time intervals in minutes. *For example, students may use a number line to determine the difference between the start time and the end time of lunch.* 3.MD.A.2 Measure the mass of objects and liquid volume using standard units of grams (g), kilograms (kg), milliliters (ml), and liters (l). Estimate the mass of objects and liquid volume using benchmarks. *For example, a large paper clip is about one gram, so a box of about 100 large clips is about 100 grams. Therefore, ten boxes would be about 1 kilogram.*

The domain is indicated at the top of the table of standards. The left column of the table contains the cluster headings. A light green coloring of the cluster heading (and codes of each of the standards within that cluster) indicates the major work of the grade. Supporting standards have no coloring. In this way, printing on a non-color printer, the standards belonging to the major work of the grade will be lightly shaded and stand distinct from the supporting standards. This color coding scheme will be followed throughout all standards K – 12. Next to the clusters are the content standards that indicate specifically what a student is to know, understand, and do with respect to that cluster. The numbering scheme for K-8 is intuitive and consistent throughout the grades. The numbering scheme for the high school standards will be somewhat different.

Example coding for grades K-8 standards:
3.MD.A.1
3 is the grade level.
Measurement and Data (**MD**) is the domain.
A is the cluster (ordered by A, B, C, etc. for first cluster, second cluster, etc.).
1 is the standard number (the standards are numbered consecutively throughout each domain regardless of cluster).

Example from the Standards' Document for 9 – 12

Taken from Integrated Math 1 Standards:

Algebra

Seeing Structure in Expressions (A.SSE)		
Cluster Headings	**Content Standards**	**Scope & Clarifications**
A. Interpret the structure of expressions.	**M1.A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.* **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.	*For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* *Tasks are limited to linear and exponential expressions, including related numerical expressions.*

The high school standards follow a slightly different coding structure. They start with the course indicator (M1 – Integrated Math 1, A1 – Algebra 1, G – Geometry, etc.), then the conceptual category (in the example below – Algebra) and then the domain (just above the table of standards it represents – Seeing Structure in Expressions). There are various domains under each conceptual category. The table of standards contains the cluster headings (see explanation above), content standards, and the scope and clarifications column, which gives further clarification of the standard and the extent of its coverage in the course. A * with a standard indicates a modeling standard (see MP4 on p.11). The color coding is light green for the major work of the grade and no color for the supporting standards.

Example coding for grades 9-12 standards:
M1.A.SSE.A.1
Integrated Math 1 (**M1**) is the course.
Algebra (**A**) is the conceptual category.
Seeing Structure in Expressions (**SSE**) is the domain.
A is the cluster (ordered by A, B, C, etc. for first cluster, second cluster, etc.).
1 is the standard number (the standards are numbered consecutively throughout each domain regardless of cluster).

Tennessee State Math Standards

The Standards for Mathematical Practice

Being successful in mathematics requires that development of approaches, practices, and habits of mind be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop within their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics.

Processes and proficiencies are two words that address the purpose and intent of the practice standards. Process is used to indicate a particular course of action intended to achieve a result, and this ties to the process standards from NCTM that pertain to problem solving, reasoning and proof, communication, representation, and connections. Proficiencies pertain to being skilled in the command of fundamentals derived from practice and familiarity. Mathematically, this addresses concepts such as adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive dispositions toward the work at hand. The practice standards are written to address the needs of the student with respect to being successful in mathematics.

These standards are most readily developed in the solving of high-level mathematical tasks. High-level tasks demand a greater level of cognitive effort to solve than routine practice problems do. Such tasks require one to make sense of the problem and work at solving it. Often a student must reason abstractly and quantitatively as he or she constructs an approach. The student must be able to argue his or her point as well as critique the reasoning of others with respect to the task. These tasks are rich enough to support various entry points for finding solutions. To develop the processes and proficiencies addressed in the practice standards, students must be engaged in rich, high-level mathematical tasks that support the approaches, practices, and habits of mind which are called for within these standards.

The following are the eight standards for mathematical practice:

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

A full description of each of these standards follows.

MP1: Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2: Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MP3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP4: Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5: Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a compass, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6: Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

MP7: Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8: Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Literacy Skills for Mathematical Proficiency

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others and analyze and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Reading

Reading in mathematics is different from reading literature. Mathematics contains expository text along with precise definitions, theorems, examples, graphs, tables, charts, diagrams, and exercises. Students are expected to recognize multiple representations of information, use mathematics in context, and draw conclusions from the information presented. In the early grades, non-readers and struggling readers benefit from the use of multiple representations and contexts to develop mathematical connections, processes, and procedures. As students' literacy skills progress, their skills in mathematics develop so that by high school, students are using multiple reading strategies, analyzing context-based problems to develop understanding and comprehension, interpreting and using multiple representations, and fully engaging with mathematics textbooks and other mathematics-based materials. These skills support Mathematical Practices 1 and 2.

Vocabulary

Understanding and using mathematical vocabulary correctly is essential to mathematical proficiency. Mathematically proficient students use precise mathematical vocabulary to express ideas. In all grades, separating mathematical vocabulary from everyday use of words is important for developing an understanding of mathematical concepts. For example, a "table" in everyday use means a piece of furniture, while in mathematics, a "table" is a way of organizing and presenting information. Mathematically proficient students are able to parse a mathematical term, definition, or theorem, provide examples and counterexamples, and use precise mathematical vocabulary in reading, speaking, and writing arguments and explanations. These skills support Mathematical Practice 6.

Speaking and Listening

Mathematically proficient students can listen critically, discuss, and articulate their mathematical ideas clearly to others. As students' mathematical abilities mature, they move from communicating through reiterating others' ideas to paraphrasing, summarizing, and drawing their own conclusions. A mathematically proficient student uses appropriate mathematics vocabulary in verbal discussions, listens to mathematical arguments, and dissects an argument to recognize flaws or determine validity. These skills support Mathematical Practice 3.

Writing

Mathematically proficient students write mathematical arguments to support and refute conclusions and cite evidence for these conclusions. Throughout all grades, students write reflectively to compare and contrast problem-solving approaches, evaluate mathematical processes, and analyze their thinking and decision-making processes to improve their mathematical strategies. These skills support Mathematical Practices 2, 3, and 4.

Algebra II | A2

Algebra II emphasizes polynomial, rational and exponential expressions, equations, and functions. This course also introduces students to the complex number system, basic trigonometric functions, and foundational statistics skills such as interpretation of data and making statistical inferences. Students build upon previous knowledge of equations and inequalities to reason, solve, and represent equations and inequalities numerically and graphically.

The major work of Algebra II is from the following domains and clusters:

- **The Real Number System**
  - Extend the properties of exponents to rational exponents.
- **Seeing Structure in Expressions**
  - Interpret the structure of expressions.
  - Use expressions in equivalent forms to solve problems.
- **Arithmetic with Polynomials and Rational Expressions**
  - Understand the relationship between zeros and factors of polynomials.
- **Reasoning with Equations and Inequalities**
  - Understand solving equations as a process of reasoning and explain the reasoning.
  - Represent and solve equations graphically.
- **Interpreting Functions**
  - Interpret functions that arise in applications in terms of the context.
- **Building Functions**
  - Build a function that models a relationship between two quantities.
- **Making Inferences and Justifying Conclusions**
  - Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Supporting work is from the following domains and clusters:

- **Quantities**
  - Reason quantitatively and use units to solve problems.
- **The Complex Number System**
  - Perform arithmetic operations with complex numbers.
  - Use complex numbers in quadratic equations.
- **Arithmetic with Polynomials and Rational Expressions**
  - Use polynomial identities to solve problems.
  - Rewrite rational expressions.
- **Creating Equations**
  - Create equations that describe numbers or relationships.
- **Reasoning with Equations and Inequalities**
  - Solve equations and inequalities in one variable.
  - Solve systems of equations.
- **Interpreting Functions**
  - Analyze functions using different representations.
- **Building Functions**
  - Build new functions from existing functions.
- **Linear, Quadratic, and Exponential Models**
  - Construct and compare linear, quadratic, and exponential models and solve problems.
  - Interpret expressions for functions in terms of the situation they model.

- **Trigonometric Functions**
    - Extend the domain of trigonometric functions using the unit circle.
    - Prove and apply trigonometric identities.
- **Interpreting Categorical and Quantitative Data**
    - Summarize, represent, and interpret data on a single count or measurement variable.
    - Summarize, represent, and interpret data on two categorical and quantitative variables.
- **Conditional Probability and the Rules of Probability**
    - Understand independence and conditional probability and use them to interpret data.
    - Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

The Real Number System (N.RN)

Cluster Headings	Content Standards	Scope & Clarifications
A. Extend the properties of exponents to rational exponents.	**A2.N.RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	*For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Quantities★ (N.Q)

Cluster Headings	Content Standards	Scope & Clarifications
A. Reason quantitatively and use units to solve problems.	**A2.N.Q.A.1** Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.	*Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

The Complex Number System (N.CN)

Cluster Headings	Content Standards	Scope & Clarifications
A. Perform arithmetic operations with complex numbers.	**A2.N.CN.A.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
A. Perform arithmetic operations with complex numbers.	**A2.N.CN.A.2** Know and use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Use complex numbers in quadratic equations.	**A2.N.CN.B.3** Solve quadratic equations with real coefficients that have complex solutions.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret the structure of expressions.	**A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.	*For example, see $2x^4 + 3x^2 – 5$ as its factors $(x^2 – 1)$ and $(2x^2 + 5)$; see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$; see $(x^2 + 4)/(x^2 + 3)$ as $((x^2+ 3) + 1)/(x^2 + 3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.* *Tasks are limited to polynomial, rational, or exponential expressions.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Use expressions in equivalent forms to solve problems.	**A2.A.SSE.B.2** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **a.** Use the properties of exponents to rewrite expressions for exponential functions.	*For example the expression 1.15^t can be rewritten as $((1.15)^{1/12})^{12t} \approx 1.012^{12t}$ to reveal that the approximate equivalent monthly interest rate is 1.2% if the annual rate is 15%.* *i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.* *ii) Tasks are limited to exponential expressions with rational or real exponents.*
	A2.A.SSE.B.3 Recognize a finite geometric series (when the common ratio is not 1), and know and use the sum formula to solve problems in context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Arithmetic with Polynomials and Rational Expressions (A.APR)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand the relationship between zeros and factors of polynomials.	**A2.A.APR.A.1** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.A.APR.A.2 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	*Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Use polynomial identities to solve problems.	**A2.A.APR.B.3** Know and use polynomial identities to describe numerical relationships.	*For example, compare (31)(29) = (30 + 1) (30 – 1) = $30^2 - 1^2$ with (x + y) (x – y) = $x^2 - y^2$.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
C. Rewrite rational expressions.	**A2.A.APR.C.4** Rewrite rational expressions in different forms.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Creating Equations★ (A.CED)

Cluster Headings	Content Standards	Scope & Clarifications
A. Create equations that describe numbers or relationships.	**A2.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.	*Include equations arising from linear and quadratic functions, and rational and exponential functions.* *Tasks have a real-world context.*
	A2.A.CED.A.2 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	*i) Tasks are limited to square root, cube root, polynomial, rational, and logarithmic functions.* *ii) Tasks have a real-world context.*

Reasoning with Equations and Inequalities (A.REI)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand solving equations as a process of reasoning and explain the reasoning.	**A2.A.REI.A.1** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	*Tasks are limited to square root, cube root, polynomial, rational, and logarithmic functions.*
	A2.A.REI.A.2 Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Solve equations and inequalities in one variable.	**A2.A.REI.B.3** Solve quadratic equations and inequalities in one variable. **a**. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.	*In the case of equations that have roots with nonzero imaginary parts, students write the solutions as a ± bi for real numbers a and b.*
C. Solve systems of equations.	**A2.A.REI.C.4** Write and solve a system of linear equations in context.	*When solving algebraically, tasks are limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to only two variables.*
	A2.A.REI.C.5 Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
D. Represent and solve equations graphically.	**A2.A.REI.D.6** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the approximate solutions using technology. ★	*Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* *Tasks may involve any of the function types mentioned in the standard.*

Functions

Interpreting Functions (F.IF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret functions that arise in applications in terms of the context.	**A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. ★	*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.* *i) Tasks have a real-world context.* *ii) Tasks may involve square root, cube root, polynomial, exponential, and logarithmic functions.*
	A2.F.IF.A.2 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	*i) Tasks have a real-world context.* *ii) Tasks may involve polynomial, exponential, and logarithmic functions.*
B. Analyze functions using different representations.	**A2.F.IF.B.3** Graph functions expressed symbolically and show key features of the graph, by hand and using technology.★ **a.** Graph square root, cube root, and piecewise defined functions, including step functions and absolute value functions. **b.** Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. **c.** Graph exponential and logarithmic functions, showing intercepts and end behavior.	*A2.F.IF.B.3a: Tasks are limited to square root and cube root functions. The other functions are assessed in Algebra 1.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Analyze functions using different representations.	**A2.F.IF.B.4** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Know and use the properties of exponents to interpret expressions for exponential functions.	*For example, identify percent rate of change in functions such as* $y = 2^x$, $y = (1/2)^x$, $y = 2^{-x}$, $y = (1/2)^{-x}$. *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.F.IF.B.5 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	*Tasks may involve polynomial, exponential, and logarithmic functions.*

Building Functions (F.BF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Build a function that models a relationship between two quantities.	**A2.F.BF.A.1** Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context. **b.** Combine standard function types using arithmetic operations.	*For example, given cost and revenue functions, create a profit function.* *For A2.F.BF.A.1a:* *i) Tasks have a real-world context.* *ii) Tasks may involve linear functions, quadratic functions, and exponential functions.*
	A2.F.BF.A.2 Know and write arithmetic and geometric sequences with an explicit formula and use them to model situations.★	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Build new functions from existing functions.	**A2.F.BF.B.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	*i) Tasks may involve polynomial, exponential, and logarithmic functions.* *ii) Tasks may involve recognizing even and odd functions.*
	A2.F.BF.B.4 Find inverse functions. **a.** Find the inverse of a function when the given function is one-to-one.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Linear, Quadratic, and Exponential Models★ (F.LE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Construct and compare linear, quadratic, and exponential models and solve problems.	**A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.F.LE.A.2 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Interpret expressions for functions in terms of the situation they model.	**A2.F.LE.B.3** Interpret the parameters in a linear or exponential function in terms of a context.	*For example, the equation $y = 5000 (1.06)^x$ models the rising population of a city with 5000 residents when the annual growth rate is 6 percent. What will be the effect on the equation if the city's growth rate was 7 percent instead of 6 percent?* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Trigonometric Functions (F.TF)

Cluster Headings	Content Standards	Scope & Clarifications
A. Extend the domain of trigonometric functions using the unit circle.	**A2.F.TF.A.1** Understand and use radian measure of an angle. **a.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **b.** Use the unit circle to find *sin* θ, *cos* θ, and *tan* θ when θ is a commonly recognized angle between 0 and 2π.	*Commonly recognized angles include all multiples nπ /6 and nπ /4, where n is an integer.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
B. Prove and apply trigonometric identities.	**A2.F.TF.B.3** Know and use trigonometric identities to to find values of trig functions. **a.** Given a point on a circle centered at the origin, recognize and use the right triangle ratio definitions of *sin* θ, *cos* θ, and *tan* θ to evaluate the trigonometric functions. **b.** Given the quadrant of the angle, use the identity $sin^2 \theta + cos^2 \theta = 1$ to find *sin* θ given *cos* θ, or vice versa.	*Commonly recognized angles include all multiples nπ /6 and nπ /4, where n is an integer.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Statistics and Probability

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards	Scope & Clarifications
A. Summarize, represent, and interpret data on a single count or measurement variable.	**A2.S.ID.A.1** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages using the Empirical Rule.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**A2.S.ID.B.2** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data.	*Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* *i) Tasks have a real-world context.* *ii) Tasks are limited to exponential functions with domains not in the integers.*

Making Inferences and Justifying Conclusions (S.IC)

Cluster Headings	Content Standards	Scope & Clarifications
A. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	**A2.S.IC.A.1** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	*For example, in a given situation, is it more appropriate to use a sample survey, an experiment, or an observational study? Explain how randomization affects the bias in a study.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.S.IC.A.2 Use data from a sample survey to estimate a population mean or proportion; use a given margin of error to solve a problem in context.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Conditional Probability and the Rules of Probability (S.CP)

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand independence and conditional probability and use them to interpret data.	**A2.S.CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.S.CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.S.CP.A.3 Know and understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Cluster Headings	Content Standards	Scope & Clarifications
A. Understand independence and conditional probability and use them to interpret data.	**A2.S.CP.A.4** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	*For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**A2.S.CP.B.5** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A* and interpret the answer in terms of the model.	*For example, a teacher gave two exams. 75 percent passed the first quiz and 25 percent passed both. What percent who passed the first quiz also passed the second quiz?* *There are no assessment limits for this standard. The entire standard is assessed in this course.*
	A2.S.CP.B.6 Know and apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)*, and interpret the answer in terms of the model.	*For example, in a math class of 32 students, 14 are boys and 18 are girls. On a unit test 6 boys and 5 girls made an A. If a student is chosen at random from a class, what is the probability of choosing a girl or an A student?* *There are no assessment limits for this standard. The entire standard is assessed in this course.*

Major content of the course is indicated by the light green shading of the cluster heading and standard's coding.

(green)	**Major Content**		**Supporting Content**