Tennessee Math Standards

Introduction

The Process

The Tennessee State Math Standards were reviewed and developed by Tennessee teachers for Tennessee schools. The rigorous process used to arrive at the standards in this document began with a public review of the then-current standards. After receiving 130,000+ reviews and 20,000+ comments, a committee composed of Tennessee educators spanning elementary through higher education reviewed each standard. The committee scrutinized and debated each standard using public feedback and the collective expertise of the group. The committee kept some standards as written, changed or added imbedded examples, clarified the wording of some standards, moved some standards to different grades, and wrote new standards that needed to be included for coherence and rigor. From here the standards went before the appointed Standards Review Committee to make further recommendations before being presented to the Tennessee Board of Education for final adoption.

The result is Tennessee Math Standards for Tennessee Students by Tennesseans.

Mathematically Prepared

Tennessee students have various mathematical needs that their K-12 education should address.

All students should be able to recall and use their math education when the need arises. That is, a student should know certain math facts and concepts such as the multiplication table, how to add, subtract, multiply, and divide basic numbers, how to work with simple fractions and percentages, etc. There is a level of procedural fluency that a student's K-12 math education should provide him or her along with conceptual understanding so that this can be recalled and used throughout his or her life. Students also need to be able to reason mathematically. This includes problem solving skills in work and non-work related settings and the ability to critically evaluate the reasoning of others.

A student's K-12 math education should also prepare him or her to be free to pursue post-secondary education opportunities. Students should be able to pursue whatever career choice, and its post-secondary education requirements, that they desire. To this end, the K-12 math standards lay the foundation that allows any student to continue further in college, technical school, or with any other post-secondary educational needs.

A college and career ready math class is one that addresses all of the needs listed above. The standards' role is to define what our students should know, understand, and be able to do mathematically so as to fulfill these needs. To that end, the standards address conceptual understanding, procedural fluency, and application.

Conceptual Understanding, Procedural Fluency, and Application

In order for our students to be mathematically proficient, the standards focus on a balanced development of conceptual understanding, procedural fluency, and application. Through this balance, students gain understanding and critical thinking skills that are necessary to be truly college and career ready.

Conceptual understanding refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.

Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly. One cannot stop with memorization of facts and procedures alone. It is about recognizing when one strategy or procedure is more appropriate to apply than another. Students need opportunities to justify both informal strategies and commonly used procedures through distributed practice. Procedural fluency includes computational fluency with the four arithmetic operations. In the early grades, students are expected to develop fluency with whole numbers in addition, subtraction, multiplication, and division. Therefore, computational fluency expectations are addressed throughout the standards. Procedural fluency extends students' computational fluency and applies in all strands of mathematics. It builds from initial exploration and discussion of number concepts to using informal strategies and the properties of operations to develop general methods for solving problems (NCTM, 2014).

Application provides a valuable context for learning and the opportunity to practice skills in a relevant and a meaningful way. As early as Kindergarten, students are solving simple “word problems” with meaningful contexts. In fact, it is in solving word problems that students are building a repertoire of procedures for computation. They learn to select an efficient strategy and determine whether the solution(s) makes sense. Problem solving provides an important context in which students learn about numbers and other mathematical topics by reasoning and developing critical thinking skills (Adding It Up, 2001).

Progressions

The standards for each grade are not written to be nor are they to be considered as an island in and of themselves. There is a flow, or progression, from one grade to the next, all the way through to the high school standards. There are four main progressions that are composed of mathematical domains/conceptual categories (see the Structure section below and color chart on the following page).

The progressions are grouped as follows:

Grade	Domain/Conceptual Category
K	Counting and Cardinality
K-5	Number and Operations in Base Ten
3-5	Number and Operations – Fractions
6-7	Ratios and Proportional Relationships
6-8	The Number System
9-12	Number and Quantity
K-5	Operations and Algebraic Thinking
6-8	Expressions and Equations
8	Functions
9-12	Algebra and Functions
K-12	Geometry
K-5	Measurement and Data
6-12	Statistics and Probability

State Standards – Mathematics Learning Progressions

<table>
<tr><th>Kindergarten</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="2">Ratios and Proportional Relationships</td><td></td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations - Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="6">Geometry</td><td colspan="3">Geometry</td><td>Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics and Probability</td></tr>
</table>

Each of the progressions begins in Kindergarten, with a constant movement toward the high school standards as a student advances through the grades. This is very important to guarantee a steady, age appropriate progression which allows the student and teacher alike to see the overall coherence of and connections among the mathematical topics. It also ensures that gaps are not created in the mathematical education of our students.

Structure of the Standards

Most of the structure of the previous state standards has been maintained. This structure is logical and informative as well as easy to follow. An added benefit is that most Tennessee teachers are already familiar with it.

The structure includes:

- **Content Standards -** Statements of what a student should know, understand, and be able to do.

- **Clusters -** Groups of related standards. Cluster headings may be considered as the big idea(s) that the group of standards they represent are addressing. They are therefore useful as a quick summary of the progression of ideas that the standards in a domain are covering and can help teachers to determine the focus of the standards they are teaching.

- **Domains -** A large category of mathematics that the clusters and their respective content standards delineate and address. For example, *Number and Operations – Fractions* is a domain under which there are a number of clusters (the big ideas that will be addressed) along with their respective content standards, which give the specifics of what the student should know, understand, and be able to do when working with fractions.

- **Conceptual Categories –** The content standards, clusters, and domains in the 9^{th}-12^{th} grades are further organized under conceptual categories. These are very broad categories of mathematical thought and lend themselves to the organization of high school course work. For example, Algebra is a conceptual category in the high school standards under which are domains such as Seeing Structure in Expressions, Creating Equations, Arithmetic with Polynomials and Rational Expressions, etc.

Standards and Curriculum

It should be noted that the standards are what students should know, understand, and be able to do; but, they do not dictate how a teacher is to teach them. In other words, the standards do not dictate curriculum. For example, students are to understand and be able to add, subtract, multiply, and divide fractions according to the standards. Although within the standards algorithms are mentioned and examples are given for clarification, how to approach these concepts and the order in which the standards are taught within a grade or course are all decisions determined by the local district, school, and teachers.

Example from the Standards' Document for K – 8

Taken from 3rd Grade Standards:

Measurement and Data (MD)	
Cluster Headings	**Content Standards**
A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve contextual problems involving addition and subtraction of time intervals in minutes. *For example, students may use a number line to determine the difference between the start time and the end time of lunch.* 3.MD.A.2 Measure the mass of objects and liquid volume using standard units of grams (g), kilograms (kg), milliliters (ml), and liters (l). Estimate the mass of objects and liquid volume using benchmarks. *For example, a large paper clip is about one gram, so a box of about 100 large clips is about 100 grams. Therefore, ten boxes would be about 1 kilogram.*

The domain is indicated at the top of the table of standards. The left column of the table contains the cluster headings. A light green coloring of the cluster heading (and codes of each of the standards within that cluster) indicates the major work of the grade. Supporting standards have no coloring. In this way, printing on a non-color printer, the standards belonging to the major work of the grade will be lightly shaded and stand distinct from the supporting standards. This color coding scheme will be followed throughout all standards K – 12. Next to the clusters are the content standards that indicate specifically what a student is to know, understand, and do with respect to that cluster. The numbering scheme for K-8 is intuitive and consistent throughout the grades. The numbering scheme for the high school standards will be somewhat different.

Example coding for grades K-8 standards:
3.MD.A.1
3 is the grade level.
Measurement and Data (**MD**) is the domain.
A is the cluster (ordered by A, B, C, etc. for first cluster, second cluster, etc.).
1 is the standard number (the standards are numbered consecutively throughout each domain regardless of cluster).

Example from the Standards' Document for 9 – 12

Taken from Integrated Math 1 Standards:

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards	Scope & Clarifications
A. Interpret the structure of expressions.	**M1.A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.* **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.	*For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* *Tasks are limited to linear and exponential expressions, including related numerical expressions.*

The high school standards follow a slightly different coding structure. They start with the course indicator (M1 – Integrated Math 1, A1 – Algebra 1, G – Geometry, etc.), then the conceptual category (in the example below – Algebra) and then the domain (just above the table of standards it represents – Seeing Structure in Expressions). There are various domains under each conceptual category. The table of standards contains the cluster headings (see explanation above), content standards, and the scope and clarifications column, which gives further clarification of the standard and the extent of its coverage in the course. A * with a standard indicates a modeling standard (see MP4 on p.11). The color coding is light green for the major work of the grade and no color for the supporting standards.

Example coding for grades 9-12 standards:
M1.A.SSE.A.1
Integrated Math 1 (**M1**) is the course.
Algebra (**A**) is the conceptual category.
Seeing Structure in Expressions (**SSE**) is the domain.
A is the cluster (ordered by A, B, C, etc. for first cluster, second cluster, etc.).
1 is the standard number (the standards are numbered consecutively throughout each domain regardless of cluster).

Tennessee State Math Standards

The Standards for Mathematical Practice

Being successful in mathematics requires that development of approaches, practices, and habits of mind be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop within their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics.

Processes and proficiencies are two words that address the purpose and intent of the practice standards. Process is used to indicate a particular course of action intended to achieve a result, and this ties to the process standards from NCTM that pertain to problem solving, reasoning and proof, communication, representation, and connections. Proficiencies pertain to being skilled in the command of fundamentals derived from practice and familiarity. Mathematically, this addresses concepts such as adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive dispositions toward the work at hand. The practice standards are written to address the needs of the student with respect to being successful in mathematics.

These standards are most readily developed in the solving of high-level mathematical tasks. High-level tasks demand a greater level of cognitive effort to solve than routine practice problems do. Such tasks require one to make sense of the problem and work at solving it. Often a student must reason abstractly and quantitatively as he or she constructs an approach. The student must be able to argue his or her point as well as critique the reasoning of others with respect to the task. These tasks are rich enough to support various entry points for finding solutions. To develop the processes and proficiencies addressed in the practice standards, students must be engaged in rich, high-level mathematical tasks that support the approaches, practices, and habits of mind which are called for within these standards.

The following are the eight standards for mathematical practice:

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

A full description of each of these standards follows.

MP1: Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2: Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MP3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP4: Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5: Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a compass, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6: Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

MP7: Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8: Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Literacy Skills for Mathematical Proficiency

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others and analyze and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies.
2. Understand and use correct mathematical vocabulary.
3. Discuss and articulate mathematical ideas.
4. Write mathematical arguments.

Reading

Reading in mathematics is different from reading literature. Mathematics contains expository text along with precise definitions, theorems, examples, graphs, tables, charts, diagrams, and exercises. Students are expected to recognize multiple representations of information, use mathematics in context, and draw conclusions from the information presented. In the early grades, non-readers and struggling readers benefit from the use of multiple representations and contexts to develop mathematical connections, processes, and procedures. As students' literacy skills progress, their skills in mathematics develop so that by high school, students are using multiple reading strategies, analyzing context-based problems to develop understanding and comprehension, interpreting and using multiple representations, and fully engaging with mathematics textbooks and other mathematics-based materials. These skills support Mathematical Practices 1 and 2.

Vocabulary

Understanding and using mathematical vocabulary correctly is essential to mathematical proficiency. Mathematically proficient students use precise mathematical vocabulary to express ideas. In all grades, separating mathematical vocabulary from everyday use of words is important for developing an understanding of mathematical concepts. For example, a "table" in everyday use means a piece of furniture, while in mathematics, a "table" is a way of organizing and presenting information. Mathematically proficient students are able to parse a mathematical term, definition, or theorem, provide examples and counterexamples, and use precise mathematical vocabulary in reading, speaking, and writing arguments and explanations. These skills support Mathematical Practice 6.

Speaking and Listening

Mathematically proficient students can listen critically, discuss, and articulate their mathematical ideas clearly to others. As students' mathematical abilities mature, they move from communicating through reiterating others' ideas to paraphrasing, summarizing, and drawing their own conclusions. A mathematically proficient student uses appropriate mathematics vocabulary in verbal discussions, listens to mathematical arguments, and dissects an argument to recognize flaws or determine validity. These skills support Mathematical Practice 3.

Writing

Mathematically proficient students write mathematical arguments to support and refute conclusions and cite evidence for these conclusions. Throughout all grades, students write reflectively to compare and contrast problem-solving approaches, evaluate mathematical processes, and analyze their thinking and decision-making processes to improve their mathematical strategies. These skills support Mathematical Practices 2, 3, and 4.

Precalculus | P

Precalculus is designed to prepare students for college level STEM focused courses. Students extend their knowledge of the complex number system to use complex numbers in polynomial identities and equations. Topics for student mastery include vectors and matrix quantities, sequences and series, parametric equations, and conic sections. Students use previous knowledge to continue progressing in their understanding of trigonometric functions and using regression equations to model quantitative data.

Precalculus includes the following domains and clusters:

- **Number Expressions**
  - Represent, interpret, compare, and simplify number expressions.
- **The Complex Number System**
  - Perform complex number arithmetic and understand the representation on the complex plane.
  - Use complex numbers in polynomial identities and equations.
- **Vectors and Matrix Quantities**
  - Represent and model with vector quantities.
  - Understand the graphic representation of vectors and vector arithmetic.
  - Perform operations on matrices and use matrices in applications.
- **Sequences and Series**
  - Understand and use sequences and series.
- **Reasoning with Equations and Inequalities**
  - Solve systems of equations and nonlinear inequalities.
- **Parametric Equations**
  - Describe and use parametric equations.
- **Conic Sections**
  - Understand the properties of conic sections and apply them to model real-world phenomena.
- **Building Functions**
  - Build new functions from existing functions.
- **Interpreting Functions**
  - Analyze functions using different representations.
- **Trigonometric Functions**
  - Extend the domain of trigonometric functions using the unit circle.
- **Graphing Trigonometric Functions**
  - Model periodic phenomena with trigonometric functions.
- **Applied Trigonometry**
  - Use trigonometry to solve problems.
- **Trigonometric Identities**
  - Apply trigonometric identities to rewrite expressions and solve equations.
- **Polar Coordinates**
  - Use polar coordinates.
- **Model with Data**
  - Model data using regression equations.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

Number Expressions (N.NE)

Cluster Headings	Content Standards
A. Represent, interpret, compare, and simplify number expressions.	**P.N.NE.A.1** Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them. **P.N.NE.A.2** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.★ **P.N.NE.A.3** Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of π and e. **P.N.NE.A.4** Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not. **P.N.NE.A.5** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

The Complex Number System (N.CN)

Cluster Headings	Content Standards
A. Perform complex number arithmetic and understand the representation on the complex plane.	**P.N.CN.A.1** Perform arithmetic operations with complex numbers expressing answers in the form $a + bi$. **P.N.CN.A.2** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **P.N.CN.A.3** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **P.N.CN.A.4** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\, i)^3 = 8$ *because* $(-1 + \sqrt{3}\, i)$ *has modulus 2 and argument 120°.*

A. Perform complex number arithmetic and understand the representation on the complex plane.	**P.N.CN.A.5** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
B. Use complex numbers in polynomial identities and equations.	**P.N.CN.B.6** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. **P.N.CN.B.7** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities (N.VM)

Cluster Headings	Content Standards
A. Represent and model with vector quantities.	**P.N.VM.A.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes *(e.g.,* $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, v*)*. **P.N.VM.A.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **P.N.VM.A.3** Solve problems involving velocity and other quantities that can be represented by vectors.
B. Understand the graphic representation of vectors and vector arithmetic.	**P.N.VM.B.4** Add and subtract vectors. **a.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **b.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **c.** Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
	P.N.VM.B.5 Multiply a vector by a scalar. **a.** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. **b.** Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lVert c\boldsymbol{v}\rVert = \lvert c\rvert v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\lvert c\rvert v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Cluster Headings	Content Standards
B. Understand the graphic representation of vectors and vector arithmetic.	**P.N.VM.B.6** Calculate and interpret the dot product of two vectors.
C. Perform operations on matrices and use matrices in applications.	**P.N.VM.C.7** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **P.N.VM.C.8** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **P.N.VM.C.9** Add, subtract, and multiply matrices of appropriate dimensions. **P.N.VM.C.10** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **P.N.VM.C.11** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **P.N.VM.C.12** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **P.N.VM.C.13** Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

Sequences and Series (A.S)

A. Understand and use sequences and series.	**P.A.S.A.1** Demonstrate an understanding of sequences by representing them recursively and explicitly.
	P.A.S.A.2 Use sigma notation to represent a series; expand and collect expressions in both finite and infinite settings.
	P.A.S.A.3 Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist. a. Determine whether a given arithmetic or geometric series converges or diverges. b. Find the sum of a given geometric series (both infinite and finite). c. Find the sum of a finite arithmetic series.

A. Understand and use sequences and series.	**P.A.S.A.4** Understand that series represent the approximation of a number when truncated; estimate truncation error in specific examples.
	P.A.S.A.5 Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Reasoning with Equations and Inequalities (A.REI)

A. Solve systems of equations and nonlinear inequalities.	**P.A.REI.A.1** Represent a system of linear equations as a single matrix equation in a vector variable. **P.A.REI.A.2** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **P.A.REI.A.3** Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, logarithmic, and rational) by graphing (solutions in interval notation if one-variable), by hand and with appropriate technology. **P.A.REI.A.4** Solve systems of nonlinear inequalities by graphing.

Parametric Equations (A.PE)

A. Describe and use parametric equations. ★	**P.A.PE.A.1** Graph curves parametrically (by hand and with appropriate technology). **P.A.PE.A.2** Eliminate parameters by rewriting parametric equations as a single equation.

Conic Sections (A.C)

A. Understand the properties of conic sections and model real-world phenomena.	**P.A.C.A.1** Display all of the conic sections as portions of a cone. **P.A.C.A.2** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **P.A.C.A.3** From an equation in standard form, graph the appropriate conic section: ellipses, hyperbolas, circles, and parabolas. Demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics. **P.A.C.A.4** Transform equations of conic sections to convert between general and standard form.

Functions

Building Functions (F.BF)

Cluster Headings	Content Standards
A. Build new functions from existing functions.	**P.F.BF.A.1** Understand how the algebraic properties of an equation transform the geometric properties of its graph. *For example, given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation (i.e., translations, stretches, reflections, and changes in periodicity and amplitude).*
	P.F.BF.A.2 Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions.
	P.F.BF.A.3 Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*
	P.F.BF.A.4 Construct the difference quotient for a given function and simplify the resulting expression.
	P.F.B F.A.5 Find inverse functions (including exponential, logarithmic, and trigonometric). **a.** Calculate the inverse of a function, $f(x)$, with respect to each of the functional operations; in other words, the additive inverse, $-f(x)$, the multiplicative inverse, *1 / f(x),* and the inverse with respect to composition, $f^{-1}(x)$. Understand the algebraic and graphical implications of each type. **b.** Verify by composition that one function is the inverse of another. **c.** Read values of an inverse function from a graph or a table, given that the function has an inverse. **d.** Recognize a function is invertible if and only if it is one-to-one. Produce an invertible function from a non-invertible function by restricting the domain.
	P.F.BF.A.6 Explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.

Interpreting Functions (F.IF)

Cluster Headings	Content Standards
A. Analyze functions using different representations.	**P.F.IF.A.1** Determine whether a function is even, odd, or neither. **P.F.IF.A.2** Analyze qualities of exponential, polynomial, logarithmic, trigonometric, and rational functions and solve real-world problems that can be modeled with these functions (by hand and with appropriate technology).★ **P.F.IF.A.4** Identify the real zeros of a function and explain the relationship between the real zeros and the x-intercepts of the graph of a function (exponential, polynomial, logarithmic, trigonometric, and rational). **P.F.IF.A.5** Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$. **P.F.IF.A.6** Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection. Describe intervals where the function is increasing or decreasing and where different types of concavity occur. **P.F.IF.A.7** Graph rational functions, identifying zeros, asymptotes (including slant), and holes (when suitable factorizations are available) and showing end-behavior. **P.F.IF.A.8** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.

Trigonometric Functions (F.TF)

Cluster Headings	Content Standards
A. Extend the domain of trigonometric functions using the unit circle.	**P.F.TF.A.1** Convert from radians to degrees and from degrees to radians. **P.F.TF.A.2** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. **P.F.TF.A.3** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **P.F.TF.A.4** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Graphing Trigonometric Functions (F.GT)

Cluster Headings	Content Standards
A. Model periodic phenomena with trigonometric functions.★	**P.F.GT.A.1** Interpret transformations of trigonometric functions. **P.F.GT.A.2** Determine the difference made by choice of units for angle measurement when graphing a trigonometric function. **P.F.GT.A.3** Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes. **P.F.GT.A.4** Find values of inverse trigonometric expressions (including compositions), applying appropriate domain and range restrictions. **P.F.GT.A.5** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **P.F.GT.A.6** Determine the appropriate domain and corresponding range for each of the inverse trigonometric functions. **P.F.GT.A.7** Graph the inverse trigonometric functions and identify their key characteristics. **P.F.GT.A.8** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Geometry

Applied Trigonometry (G.AT)

Cluster Headings	Content Standards
A. Use trigonometry to solve problems. ★	**P.G.AT.A.1** Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles. **P.G.AT.A.2** Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **P.G.AT.A.3** Derive and apply the formulas for the area of sector of a circle. **P.G.AT.A.4** Calculate the arc length of a circle subtended by a central angle.

Cluster Headings	Content Standards
A. Use trigonometry to solve problems. ★	**P.G.AT.A.5** Prove the Laws of Sines and Cosines and use them to solve problems. **P.G.AT.A.6** Understand and apply the Law of Sines (including the ambiguous case) and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Trigonometric Identities (G.TI)

Cluster Headings	Content Standards
A. Apply trigonometric identities to rewrite expressions and solve equations.★	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle. **P.G.TI.A.2** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Polar Coordinates (G.PC)

Cluster Headings	Content Standards
A. Use polar coordinates.	**P.G.PC.A.1** Graph functions in polar coordinates. **P.G.PC.A.2** Convert between rectangular and polar coordinates. **P.G.PC.A.3** Represent situations and solve problems involving polar coordinates.★

Statistics and Probability

Model with Data★ (S.MD)

Cluster Headings	Content Standards
A. Model data using regressions equations.	**P.S.MD.A.1** Create scatter plots, analyze patterns, and describe relationships for bivariate data (linear, polynomial, trigonometric, or exponential) to model real-world phenomena and to make predictions. **P.S.MD.A.2** Determine a regression equation to model a set of bivariate data. Justify why this equation best fits the data.

A. Model data using regressions equations.	**P.S.MD.A.3** Use a regression equation, modeling bivariate data, to make predictions. Identify possible considerations regarding the accuracy of predictions when interpolating or extrapolating.